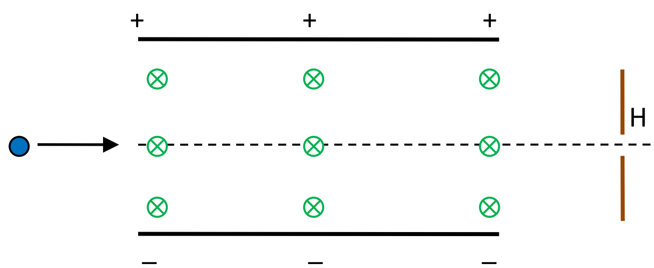
K.A. Tsokos

Sample Paper 2 long 20 mark questions

Q1 SL&HL

A beam of singly ionized atoms of the same element enters the region in between two parallel, oppositely charged plates in vacuum. The atoms have a range of speeds. A uniform magnetic field *B* of magnetic flux density 0.40 T is established in between the plates, directed into the page. The potential difference between the plates is 2.50 kV and the plates are 8.0 mm apart. The initial direction of the beam is aligned with a small hole H beyond the plates.



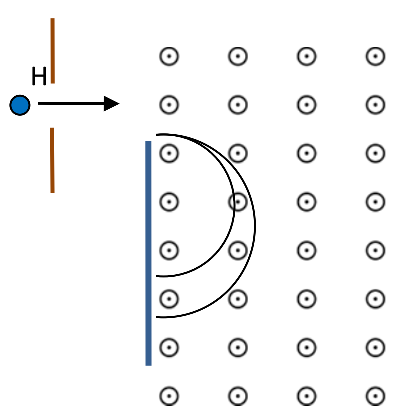
not to scale

(a) (i) Determine the electric field in between the plates. [1]

(ii) Explain why all the atoms that emerge through H have the same speed. [3]

(iii) Show that the common speed at H is about $7.8\times10^{5}\ \text{m s}^{-1}$. [1]

(b) The atoms in (a) that have gone through H enter a new region of magnetic field as shown. The magnetic flux density is 0.50 T and is directed out of the plane of the page.



The atoms are bent into two circular paths of different radius.

(i) Show that the radius of the circular path of charged particle in a magnetic field is given by $R=\dfrac{mv}{eB}$. [1]

(ii) State what is meant by isotopes. [2]

(iii) Outline why the presence of more than one path is evidence for isotopes. [2]

(c) The beam consists of stable atoms of neon of charge +*e*. The path of least radius corresponds to ${}^{20}_{10}\text{Ne}$.

(i) Show that this radius is about 0.3 m. [2]

(ii) Estimate the mass number of the isotope corresponding to a radius of 0.36 m. [1]

(d) ${}^{23}_{10}\text{Ne}$ is an unstable isotope of neon. ${}^{23}_{10}\text{Ne}$ decays into sodium (Na) by beta minus decay.

(i) Radioactive decay is described as random and spontaneous. State what this means. [2]

(ii) Write down the decay equation. [2]

(e) The **atomic** mass of ${}^{23}_{10}\text{Ne}$ is M_{Ne} = 22.9945 u and the **atomic** mass for Na is M_{Na} = 22.9898 u.

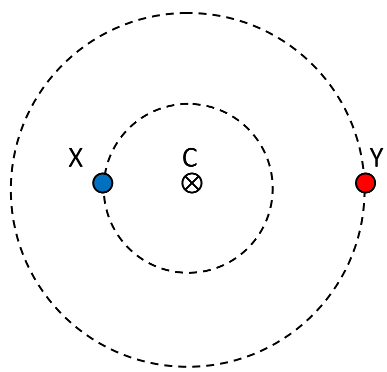
Determine the energy released in the decay. [3]

Markscheme

Q1			
a	i	$E=\frac{V}{d}=\frac{2.5\times10^{3}}{8.0\times10^{-3}}=3.125\times10^{5}\approx3.1\times10^{5}$ N C^{-1} ✓	[1]
a	ii	The atoms that will go through H must be undeflected ✓ So $qE=qvB$ ✓ $v=\frac{E}{B}$ i.e. speed is unique ✓	[3]
a	iii	$v=\frac{3.125\times10^{5}}{0.40}=7.813\times10^{5}\approx7.8\times10^{5}$ m s^{-1} ✓	[1]
b	i	$qvB=\frac{mv^{2}}{R}$ hence result ✓	[1]
b	ii	Atoms of the same element/same number of protons ✓ But different number of neutrons ✓	[2]
b	iii	Different paths are due to different mass since $R=\frac{mv}{eB}$ and v, q and B are the same ✓ Different mass can only be due to extra neutrons since the proton number is the same /same element	[2]
c	i	$R=\frac{20\times1.66\times10^{-27}\times7.813\times10^{5}}{1.6\times10^{-19}\times0.50}$ ✓ $R=0.324\approx0.3$ m ✓	[2]
c	ii	$\frac{0.36}{0.324}\times20=22.2\approx22$ ✓	[1]
d	i	Random: it cannot be predicted which nucleus and when will decay✓ Spontaneous: the rate of decay cannot be influenced/changed✓	[2]
d	ii	${}^{23}_{10}\text{Ne}\rightarrow{}^{23}_{11}\text{Na}+e^{-}+\bar{\nu}$ Correct numbers for Na ✓ Presence of antineutrino ✓	[2]
e		$Q=\Delta mc^{2}=(\bar{M}_{\text{Ne}}-\bar{M}_{\text{Na}}-m_{e})$ where the bar denotes **nuclear** masses ✓ $\Delta m=(M_{\text{Ne}}-10m_{e})-((M_{\text{Na}}-11m_{e})+m_{e})=(M_{\text{Ne}}-M_{\text{Na}})$ ✓ $Q=(22.9945-22.9898)\times931.5=4.4$ MeV ✓	[3]

Q2 HL only

(a) A binary star system consists of two stars, X and Y, that revolve around a common centre C. The stars are always diametrically opposite each other. The star system is far from all other masses.



Star X has mass M_X, orbit radius R_X and speed v_X. Star Y has mass M_Y, orbit radius R_Y and speed v_Y.

(i) Explain why $v_Y > v_X$. [2]

(ii) Suggest why the total linear momentum of the system is zero. [3]

(iii) Explain why $M_X > M_Y$. [2]

(b) Show that

(i) the speed of star X is $v_X = \sqrt{\dfrac{GM_Y R_X}{(R_X + R_Y)^2}}$, [2]

(ii) the total energy of the system is $E_T = -\dfrac{1}{2}\dfrac{GM_X M_Y}{R_X + R_Y}$. [3]

(c) The star system loses energy by radiating gravitational waves. Suggest the effect of this on the separation of the stars. [2]

(d) Star X emits light of wavelength 440 nm and star Y light of wavelength 660 nm. The ratio of the masses of the stars is $\dfrac{M_X}{M_Y} = 2$. Diagrams 1 and 2 show the positions of the stars at two different times.

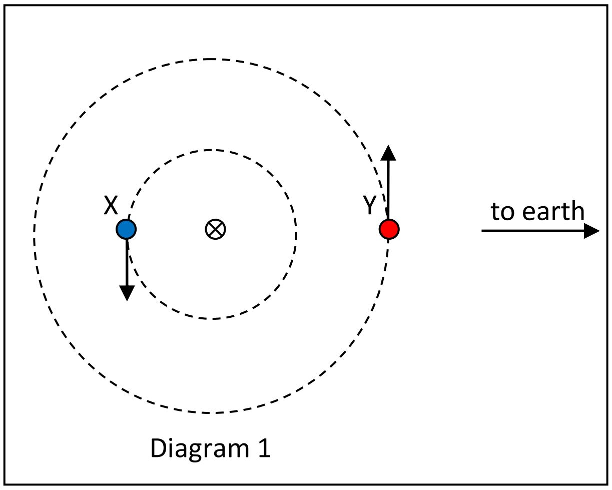


Diagram 1

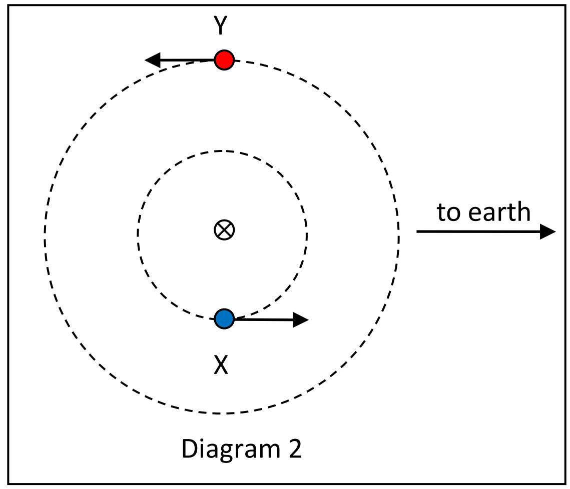


Diagram 2

Light from the stars is received on earth and is passed through a diffraction grating. The diagram shows the spectral lines when the stars are in the positions shown in Diagram 1.

When the stars are in the positions of Diagram 2,

(i) estimate the ratio $\frac{\Delta\lambda_Y}{\Delta\lambda_X}$ of the shifts in wavelength observed at earth. [3]

(ii) draw, on the diagram, the spectral lines of the stars. [3]

Markscheme

Q2			
a	i	Y covers a larger distance ✓ In the same time ✓	[2]
a	ii	The total momentum must be constant since there are no external forces ✓ If the momenta of X and Y are not equal in magnitude, in diagram 1 the total momentum is directed up or down ✓ In that case in diagram 2 the total momentum would change direction which is impossible ✓ Hence the conclusion	[3]
a	iii	$M_X v_X = M_Y v_Y \Rightarrow \frac{M_X}{M_Y} = \frac{v_Y}{v_X}$ ✓ $\frac{M_X}{M_Y} > 1$ ✓	[2]
b	i	$\frac{M_X v_X^2}{R_X} = \frac{GM_X M_Y}{(R_X + R_Y)^2}$ ✓ Simplifications to get result ✓	[2]
b	ii	KE of X and Y are $\frac{1}{2}M_X v_X^2 + \frac{1}{2}M_Y v_Y^2 = \frac{1}{2}\frac{GM_X M_Y R_Y}{(R_X+R_Y)^2} + \frac{1}{2}\frac{GM_X M_Y R_X}{(R_X+R_Y)^2} = \frac{1}{2}\frac{GM_X M_Y}{(R_X+R_Y)}$ ✓ PE of X and Y are $-\frac{GM_X M_Y}{(R_X+R_Y)}$ ✓ Total energy is $\frac{1}{2}\frac{GM_X M_Y}{(R_X+R_Y)} - \frac{GM_X M_Y}{(R_X+R_Y)} = -\frac{1}{2}\frac{GM_X M_Y}{(R_X+R_Y)}$ ✓	[3]
c		The total energy decreases (becomes more negative) ✓ So separation decreases ✓	[2]
d	i	$\frac{\Delta\lambda_Y}{\Delta\lambda_X} = \frac{\lambda_Y \frac{v_Y}{c}}{\lambda_X \frac{v_X}{c}} = \frac{\lambda_Y}{\lambda_X}\frac{v_Y}{v_X}$ ✓ $\frac{v_Y}{v_X} = \frac{M_X}{M_Y} \Rightarrow \frac{\Delta\lambda_Y}{\Delta\lambda_X} = \frac{\lambda_Y}{\lambda_X}\frac{M_X}{M_Y}$ ✓ $\frac{\Delta\lambda_Y}{\Delta\lambda_X} = \frac{660}{440} \times 2 = 3$ ✓	[3]
d	ii	(Original lines shown dashed)  Blue blueshifted ✓ Red redshifted ✓ Redshift 3 times larger ✓	[3]

Q3 (HL only)

(a) Bohr suggested that in hydrogen electrons can only exist in orbits such that $mv_n r_n = n\dfrac{h}{2\pi}$.

Outline the problem in the Rutherford model of the atom that this suggestion solves. [2]

(b) A consequence of Bohr's suggestion is that the radius of the electron's orbit in the n^{th} state of hydrogen is given by $r_n = a_0 n^2$ where $a_0 = 0.529\times10^{-10}$ m.

(i) Show that the period of revolution T_n of an electron in the n^{th} state of hydrogen is given by

$$T_n = \frac{4\pi^2 a_0^2 m}{h} n^3 .$$ [3]

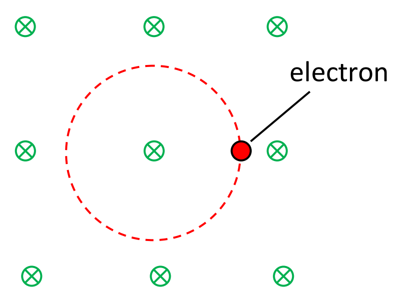
(ii) Calculate T_{20}. [2]

(iii) The frequency of the photon emitted in the transition from n = 21 to n = 20 is $f_{21\to20}$.

Estimate the product $T_{20}\times f_{21\to20}$ commenting on the result. [4]

(iv) Show that $T_n^2 = kr_n^3$ where k is a constant. [2]

(c) An electron moves in a circular path in a region of uniform magnetic field of magnetic flux density B.



The field is directed into the plane of the page.

(i) Draw, on the diagram, an arrow to indicate the velocity of the electron at the position shown. [1]

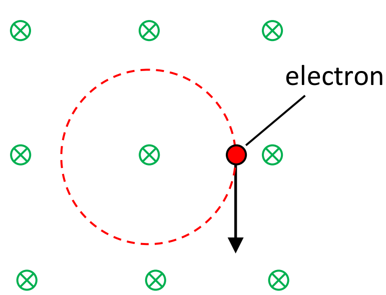
(ii) The condition $mvr = n\dfrac{h}{2\pi}$ applies to the motion of this electron. Show that $r = \sqrt{\dfrac{nh}{2\pi eB}}$. [3]

(iii) Predict that the magnetic flux Φ through the area of the electron's orbit is given by

$$\Phi = \frac{h}{2e} n .$$ [2]

(iv) Calculate the minimum value of the magnetic flux. [1]

Markscheme

Q3			
a		The electron is accelerated and so would radiate energy ✓ Eventually plunging into the nucleus ✓	[2]
b	i	From $mv_n r_n = n\frac{h}{2\pi}$ we get $v_n = n\frac{h}{2\pi m r_n} = n\frac{h}{2\pi m a_0 n^2} = \frac{h}{2\pi m a_0 n}$ ✓ $T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi a_0 n^2}{\frac{h}{2\pi m a_0 n}}$ ✓ $T_n = \frac{4\pi^2 a_0^2 m}{h} n^3$ ✓	[3]
b	ii	$T_{20} = \frac{4\pi^2 \times (0.59\times 10^{-10})^2 \times 9.1\times 10^{-31}}{6.63\times 10^{-34}} \times 20^3$ ✓ $T_2 = 1.528\times 10^{-13} \approx 1.5\times 10^{-13}$ s ✓	[2]
b	iii	$hf_{21\to 20} = \left(\frac{13.6}{20} - \frac{13.6}{21}\right) \times 1.6\times 10^{-19}$ ✓ $f_{21\to 20} = 7.81\times 10^{12}$ Hz ✓ $f_{21\to 20} T_{20} = 7.81\times 10^{12} \times 1.528\times 10^{-13} = 1.19 \approx 1$ ✓ It appears that for large n, the frequency of the photon emitted in the transition from the state n + 1 to the state n is about the same as the frequency of revolution in the state n ✓	[4]
b	iv	$T_n = \frac{4\pi^2 a_0^2 m}{h} n^3 \Rightarrow T_n^2 \propto n^6$ ✓ $r_n = a_0 n^2 \Rightarrow r_n^3 \propto n^6$ ✓ Result follows	[2]
c	i	 Arrow as shown ✓	[1]
c	ii	$evB = m\frac{v^2}{r}$ and so $r = \frac{mv}{qB}$ ✓ From $mvr = n\frac{h}{2\pi}$ we find $v = n\frac{h}{2\pi m r}$ ✓ $r = \frac{m}{eB} \times n\frac{h}{2\pi m r}$ ✓ i.e.	[3]

		$r=\sqrt{\dfrac{nh}{2\pi eB}}$	
c	iii	$\Phi=\pi r^2 B$ ✓ $\Phi=\pi\dfrac{hn}{2\pi eB}B=\dfrac{h}{2e}n$ ✓	[2]
c	iv	$\dfrac{h}{2e}=\dfrac{6.34\times10^{-34}}{2\times1.6\times10^{-19}}=2.07\times10^{-15}$ Wb	[1]